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Duality theory in empirical work, revisited

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Abstract
We compute a pseudo-dataset by Monte Carlo simulations featuring important characteristics of US agriculture, such that the initial technology parameters are known, and employing widely used datasets for calibration. Then, we show the usefulness of this calibration by applying the duality theory approach to datasets bearing as sources of noise only the aggregation of technologically heterogeneous firms. Estimation recovers initial parameters with reasonable accuracy. These conclusions are expected, but the proposed calibration sets the basis for analysing the performance of duality theory in empirical work when datasets have more observed and unobserved sources of noise, as those faced by practitioners.

Disciplines
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DUALITY THEORY IN EMPIRICAL WORK, REVISITED

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Duality theory in empirical work, revisited

The Neoclassical theory of production establishes that a competitive firm's optimization problem is characterized by a dual relationship between the value function (profit, cost, or revenue function) and the underlying production function (e.g., Mas-Colell, Winston, and Green 1995). In particular, the functional form of the production function implies a specific functional form of the profit, cost, or revenue function. Alternatively, for a given functional form used to approximate the firm's value function, there exists an underlying production function wherein the value function parameters appear in a specific way.

This dual relationship has been widely used in empirical work as a tool to estimate production parameters without explicitly specifying the parametric form of the production function. Shumway (1995) and Fox and Kivanda (1994) list more than one hundred applications of duality theory in nine agricultural economics journals only between 1974 and 1992. Typically, empirical studies consist of

i. Approximating the value function (profit, cost, or revenue function) by a parametric functional form.

ii. Deriving a set of input demand and output supply equations by applying Shephard's lemma or Hotelling's lemma.

iii. Using econometric methods to jointly estimate the parameters of the system described in (ii). In some instances, value function parameters are estimated together with those of the input demand and output supply system.

iv. Using estimated parameters from (iii) to draw conclusions about, for example, substitution elasticities, price elasticities, and/or returns to scale.

Conclusions from duality applications may be influenced by the choice of specific functional forms. As a result, there is a vast literature analyzing and testing theoretical properties such as monotonicity and curvature, with Gagné and Ouellete (1998), Terrell (1996), and Diewert and Wales (1987) as prominent examples. Also, a large number of studies focus on investigating the most preferable (flexible) functional forms for empirical purposes (Guilkey, Lovell and Sickles 1983; Dixon, Garcia and Anderson 1987; Thompson and Langworthy 1989). These studies used simulated datasets that assumed the basic tenets underlying duality theory, including perfect competition, profit maximizing behavior, and certainty. Therefore, they only consider empirical deviations from duality theory assumptions stemming from the choice of functional form; in other words, the datasets arising from the data generating process (DGP) are free from problems commonly encountered in data available to practitioners,[1] preventing these studies from evaluating the effects of the mentioned problems on the ability of the approach to recover production parameters.

Similarly, early attempts to analyze empirical properties of duality theory include Burgess (1975), Appelbaum (1978), and Lusk et al. (2002). With the exception of Lusk et al. (2002), they fail to identify the source of the discrepancy between conclusions from the primal and dual approaches. There are two reasons. They use real-world data with unknown production parameters, and furthermore, they use non-dual functional forms.

In general, the primal and dual approaches cannot provide perfectly matching parameters if a self-dual function is not used, or if there is noise. The presence of random noise weakens the dual-

[1] Examples of such problems include optimization under uncertainty; prediction errors in prices and quantities of variable netputs; omitted variable netputs; output and input data aggregation; measurement errors in the observed variables; and endogenous output and input prices. Other potential sources of noise are the incorrect specification of variable inputs as quasi-fixed, and vice versa.

primal relationship, and it is not clear whether the imprecision in the dual model caused by the noise is amplified or reduced in the primal model. The present study explores this issue, and provides evidence suggesting that the noise amplifies the imprecision in the primal model. Given that all of the real world-data sets which have been used for duality applications depict considerable amounts of noise, the scant attention paid to the impact of noise on the empirical performance of duality theory is nothing but surprising.

As a first and critical step to analyze the implications of noisy data for dual-primal relationships in practice, the contribution of the present study consists of showing how to generate a pseudo-dataset by Monte Carlo simulations calibrated to replicate key features of U.S. agriculture; more precisely, a panel of price and quantity variables based on a set of profit-maximizing firms with heterogeneous technology, which decide the quantity of variable netputs, facing variable netput prices, and conditioned on a set of quasi-fixed netputs.

A set of initial or simulated parameters is calibrated such that variables' behavior exhibit the main features of observed and widely used agricultural datasets. In particular, the one constructed and maintained by Eldon Ball for U.S. input/output price and quantities (USDA-ERS), the USDA Agricultural Resource Management Survey database (USDA-ARMS), the U.S. Agricultural Census database (USDA-NASS), and the Chicago Mercantile Exchange (CME) future prices database. We chose the first dataset because it is publicly available and it has been used for applications of duality theory in several widely cited papers (Ball 1985; Ball 1988; Baffes and Vasavada 1989; Shumway and Lim 1993; Chambers and Pope 1994). The remaining three datasets provide useful information for calibrating cross-sectional and time-series parameters, as well as noise directly observed (e.g., price variability and length of time series) and unobserved (e.g., firm heterogeneity). We adopt the criteria of calibrating parameter values to favor recovery of initial production parameters, especially for those that are unobservable.

To illustrate the usefulness of the simulated dataset in the context of the duality theory approach, we aggregate the pseudo-data set over heterogeneous firms, construct time series of prices and quantities, and estimate the set of netput elasticities with respect to prices in the dual model.[2] By comparing estimated elasticities with the underlying and known (primal) parameters of the pseudo-data, we show the performance of duality theory in empirical work when the only source of deviation from duality theory assumptions is the aggregation over heterogeneous firms. Another legitimate exercise constitutes the evaluation of the ability of the duality theorem to recover the dual parameters based on the parameter estimation performed on the primal problem, i.e., the reverse direction: dual-primal. This is an exercise as interesting and relevant as the one pursued here. However, due to space limitations, the present study only addresses the dual-primal direction. Our choice is based on the fact that the estimation performed on the dual parameters has been the most preferred approach in empirical applications, as our literature review shows. Certainly, analyzing the performance of the primal-dual direction constitutes an important topic for future research.

These pseudo-data are intended to serve as the basis or first step to conduct a thorough analysis of the performance of the dual approach in empirical work. Importantly, they can also be used in other applications, especially when knowledge of the underlying parameters is useful for the

[2] For a complete discussion of aggregation properties over firms of flexible functional forms, see Chambers (1988). In this study, we generate a panel data of observations across firms and over time. We focus here on the properties of duality theory applications using time series data. The analysis of applications with cross-sectional data is as relevant as the one pursued here, but we leave it for future research. The properties of duality theory using panel data can also be examined with the data generated, but panel studies are less frequent in the literature because such datasets are not as readily available.

objectives of the study. Datasets used by practitioners in empirical applications are noisier relative to the pseudo-dataset generated here; however, these sources of observable and unobservable noise (listed in footnote #1) are straightforward to incorporate to the pseudo-dataset, and can be calibrated using the aforementioned agricultural datasets. As these issues have not been addressed in depth in previous studies, future work in this direction constitutes relevant contributions to the literature.

The layout of the paper is as follows. After providing the theoretical framework, we proceed to describe the creation of the simulated the pseudo-dataset, including an explanation of the DGP used for that purpose. Then, model parameters are estimated using these data, and finally a comparison is made between the simulated and the estimated parameters in the results section. Concluding remarks are presented in the last section.

Model of a Single Firm

Consider a producer who chooses the level of netputs[3] to maximize profits. The producer's problem can be described as follows:

$$max_{[\boldsymbol{y},y_0]}\{\boldsymbol{p}'\boldsymbol{y}+y_0\} \quad (1)$$

where $\boldsymbol{y}$ is a choice vector of n variable netput quantities, $\boldsymbol{p}$ is a vector of n variable netput prices normalized by p_0 or the price of the numeraire commodity y_0. The augmented vector $[y_0, \boldsymbol{y}', \boldsymbol{K}']$ is referred to as the production plan of the production possibilities set S which is a subset of

[3] We use the standard definition of netput, where a positive value represents a net output and a negative value represents a net input.

R^{1+n+m}, with m equal to the number of quasi-fixed netputs (denoted as the vector $\boldsymbol{K}$) that constrain the production possibilities set.[4]

Jorgenson and Lau (1974) showed existence of a one-to-one correspondence between the set S (with properties described in footnote 3) and a production function G defined as: [5]

$$G(\boldsymbol{y}, \boldsymbol{K}) = -max\,\{y_0 / \,[y_0, \boldsymbol{y}', \boldsymbol{K}'] \in S\} \tag{2}$$

We follow the convention that $max\{\emptyset\} = -\infty$, where $\{\emptyset\}$ is defined as the empty set, such that the value of the production function is positive infinity if a production plan is not feasible. The set of quasi-fixed netputs that constrains the set S also restricts the production function G. The maximization problem can be rewritten as:

$$max_{[\mathbf{y}]}\{\boldsymbol{p}'\boldsymbol{y} - G(\boldsymbol{y}, \boldsymbol{K})\} \tag{3}$$

The solution to problem (3) is a set of netput demand equations $\boldsymbol{y}^*(\boldsymbol{p}, \boldsymbol{K})$ and a restricted profit function $\pi_R(\boldsymbol{p}, \boldsymbol{K})$ which are dependent on the vector of normalized netput prices and the vector of quasi-fixed netputs.

Lau (1976) derived the relationships between the Hessian of the production function $G(\boldsymbol{y}, \boldsymbol{K})$ and the Hessian of the restricted profit function $\pi_R(\boldsymbol{p}, \boldsymbol{K})$ under the assumption of convexity and twice continuous differentiability of both functions. Omitting the arguments of each function to simplify notation, the identities are as follows:

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[4] The properties of the set S include: i) the origin belongs to S; ii) S is closed; iii) S is convex; iv) S is monotonic with respect to y_0; and v) non-producibility with respect to at least one variable input, which implies at least one commodity is freely disposable and can only be a net input in the production process (a primary factor of production).

[5] The properties of the production function G are: i) the domain is a convex set of R^{n+m} that contains the origin; ii) the value of G at the origin, say $G(0)$, is non-positive; iii) G is bounded; iv) G is closed; and v) G is convex in $\{\boldsymbol{y}, \boldsymbol{K}\}$. Convexity (instead of concavity) is required because the convention used in Lau (1974) to define the production function contains a negative sign, that is: $y_0 = -G(\boldsymbol{y}, \boldsymbol{K})$.

$$\begin{bmatrix} \frac{\partial^2 \pi_R}{\partial \boldsymbol{p}^2} & \frac{\partial^2 \pi_R}{\partial \boldsymbol{p} \partial \boldsymbol{K}} \\ \frac{\partial^2 \pi_R}{\partial \boldsymbol{K} \partial \boldsymbol{p}} & \frac{\partial^2 \pi_R}{\partial \boldsymbol{K}^2} \end{bmatrix} \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$B_{11} = \left[\frac{\partial^2 G}{\partial \boldsymbol{y}^2}\right]^{-1} \tag{4}$$

$$B_{12} = (B_{21})' = -\left[\frac{\partial^2 G}{\partial \boldsymbol{y}^2}\right]^{-1} \left[\frac{\partial^2 G}{\partial \boldsymbol{y} \partial \boldsymbol{K}}\right]$$

$$B_{22} = -\left[\frac{\partial^2 G}{\partial \boldsymbol{K}^2}\right] - \left[\frac{\partial^2 G}{\partial \boldsymbol{K} \partial \boldsymbol{y}}\right] B_{11} \left[\frac{\partial^2 G}{\partial \boldsymbol{y} \partial \boldsymbol{K}}\right]$$

By defining, in a similar fashion, the production function Hessian sub-matrices as A_{ij}, the identities can be rewritten in the following more compact form:

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} [A_{11}]^{-1} & -[A_{11}]^{-1}[A_{12}] \\ -[A_{21}][A_{11}]^{-1} & -[A_{22}] - [A_{21}][A_{11}]^{-1}[A_{12}] \end{bmatrix} \tag{5}$$

The Hessian relationships allow us to "transform" the estimated parameters of the restricted profit function into parameters of the underlying production function, and then compare these transformed parameters with the initial parameters of the production function. More precisely, as the first derivatives of the restricted profit function ($\pi_R(\boldsymbol{p}, \boldsymbol{K})$) with respect to netput prices and quasi-fixed netputs produce the system of input demands and output supplies (Hotelling's Lemma), whose parameters we estimate econometrically, this system's first-derivatives are all that is required to obtain the Hessian of the profit function (which end-up containing as its entries the marginal effects with respect to netput prices and quasi-fixed inputs). We then transform marginal effects into elasticities and compare against their "initial" counterparts using the Hessian identities.

Ultimately, the Hessian identities allow us to assess how precisely demand and supply elasticities are estimated.

To make this problem operational, we assume a quadratic flexible form for the production function $G(\boldsymbol{y}_{ft}, \boldsymbol{K}_{ft}; \boldsymbol{\alpha}_f)$:

$$G(.) = \boldsymbol{y}'_{ft} A_{1f} + \boldsymbol{K}'_{ft}\, A_{2f} + \frac{1}{2} \boldsymbol{y}'_{ft} A_{11f} \boldsymbol{y}_{ft} + \boldsymbol{y}'_{ft} A_{12f} \boldsymbol{K}_{ft} + \frac{1}{2} \boldsymbol{K}'_{ft} A_{22f} \boldsymbol{K}_{ft} - \psi_{ft} \qquad (6)$$

where A_{1f} and A_{2f} are $(n \times 1)$ and $(m \times 1)$ vectors of $\alpha_{i,f}$ coefficients, A_{11f} is a symmetric and nonsingular $(n \times n)$ matrix, A_{12f} and A_{22f} are $(n \times m)$ and $(m \times m)$ matrices of firm f, and term ψ_{ft} is a mean-zero, firm- and time-specific production shock. Submatrices $A_{11f},\ A_{12f}$ and A_{22f} form a symmetric and positive semi-definite $\big((n + m) \times (n + m)\big)$ matrix A_f of $\alpha_{ij,f}$ coefficients.[6] We collectively denote all $\alpha_{i,f}$ and $\alpha_{ij,f}$ coefficients by $\boldsymbol{\alpha}_f$.

The quadratic functional form is selected for three reasons. First, it is self-dual—the functional form of the constrained or unconstrained profit function consistent with this production function is also quadratic. This favors recovery of the starting production parameters because the estimation is free from errors arising from functional form specification. Second, the Hessian matrices of both the production and profit functions are only functions of parameters; this proves to be useful because the comparison of the profit and production function Hessians does not depend on the set of model variables at which Hessians are evaluated. Third, the normalized quadratic profit function is extensively used in empirical analysis (Schuring, Huffman and Fan 2011; Arnade and Kelch

[6] Positive semi-definiteness is required because of the convention used in Lau (1976) that $y_0 = -G(\boldsymbol{y}, \boldsymbol{K})$.

2007; Lusk et al. 2002; Lim and Shumway 1993; Huffman and Evenson 1989; Thompson and Langworthy 1989).[7]

Simulation of Panel Data

The DGP considers variability of prices and quantities over time within three regions composed of heterogeneous firms. Heterogeneity across regions is assumed to be higher than heterogeneity of firms within each region. The simulated DGP consists of a panel of F = 10,000 farms, in R = 3 regions, over T = 50 years ($R \times F \times T$ = 1.5 million) for each element of the vector $[\boldsymbol{y}_{ft}, \boldsymbol{p}_{ft}, \boldsymbol{K}_{ft}; \boldsymbol{a}_f^*]$, where f and t index firms and time periods (years) respectively,[8] conditional on the initial (*) value of the production parameter set $\boldsymbol{a}_f^*$.

The parameter vector $\boldsymbol{a}_f^*$ does not depend on time, which implies that technology remains unchanged from period one through T. This assumption favors the recovery of starting production parameters because the estimation is free from misspecification that may arise from the evolution of technology over time. This is equivalent to postulating a specific form of netput technological change and proceeding to estimation by exactly specifying its form as if the econometrician knew it with certainty. A different model specification of the mentioned technical change would only add noise to the estimation process. The study of productivity changes over time, their measurement, and their effects on the recovery of production parameters is a relevant research topic which is beyond the scope of this paper and is left for future research.

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[7] The normalized quadratic, however, has the drawbacks of not being invariant to the choice of the numeraire good (Mahmud, Robb, and Scarth 1987) and of generating an asymmetry in the structure of the technology and the underlying objective functions (Diewert and Wales 1987). The symmetric McFadden quadratic functional form does not pose these problems but is not self-dual; hence, it is not appropriate for this study.

[8] The simulated panel corresponds to roughly about one-fifth of the quantity of farms in a given state of the Corn Belt, Lake States and Northern Plains regions in the U.S. (Corn Belt states: IA, IL, IN, MO, OH; Lake States: MI, MN, WI; and Northern Plains states: KS, ND, NE, SD). State-level time-series datasets with information on prices and quantities of agricultural outputs and inputs are available for no more than 50 years in the U.S.

Figure 1 shows the data simulation process. We start by creating the variables conditioning the firm's decisions problem in (3). First, we generate the set of starting production function parameters $\boldsymbol{a}_f^*$ and the quasi-fixed netputs $\boldsymbol{K}_{ft}^*$. Second, conditioning on these values, we draw normalized variable netput prices $\boldsymbol{p}_{ft}^{**}$. Data generation of $\boldsymbol{a}_f^*$, $\boldsymbol{K}_{ft}^*$, and $\boldsymbol{p}_{ft}^{**}$ are explained in subsections *A* through *C*. Third, we solve a profit maximization problem to obtain the variable netput quantities $\boldsymbol{y}_{ft}^{**}$ (subsection *D*). This study focuses on time-series estimation and therefore we aggregate variables across heterogeneous firms before proceeding to estimation of the profit function parameters that are then transformed to production function parameters by Hessian identities (subsection *E*). The result is a set of estimated production parameters denoted as $\breve{\boldsymbol{\alpha}}_{\boldsymbol{f}}$.

A. Simulation of initial production function parameters: $\boldsymbol{a}_{\boldsymbol{f}}^*$

The value of $\boldsymbol{a}_f^*$ characterizes the firm's technology and is unobserved, making its simulation more challenging. From (6), $\boldsymbol{a}_f^*$ consists of the submatrices A_{1f}, A_{2f}, and A_f (formed in turn by A_{11f}, A_{12f} and A_{22f}). As we mentioned above, firm heterogeneity exists both within and across regions, such that technology is more similar between firms in the same region than across regions. Hence, we select values of the elements of $\boldsymbol{\alpha}$ for a "generic" firm such that the symmetric $\big((n+m)\times(n+m)\big)$ matrix *A* is positive-semidefinite. To induce variation across regions we obtain "regional" $\boldsymbol{\alpha}_{\boldsymbol{r}}$ sets as deviations from $\boldsymbol{\alpha}$. Then, firm heterogeneity within a region comes from generating parameters in the firm-specific set $\boldsymbol{\alpha}_{\boldsymbol{f}}$ as deviations from their corresponding regional $\boldsymbol{\alpha}_{\boldsymbol{r}}$. To assure the matrix A_f and its inverse are positive-semidefinite, we draw the entries of the upper triangular matrix C_f, the Cholesky decomposition of matrix $\left(A_f\right)^{-1}$, such that the latter is formed as the matrix product $C_f'C_f$ (Hamilton 1994, p. 147).

The size, dispersion, and skewness of the elements in $\boldsymbol{\alpha}_f$ determine the size, dispersion, and skewness of the netput quantity variables, $\boldsymbol{y}_{ft}^{*}$, according to the first-order conditions (FOCs) of the firm's optimization problem. Therefore, these elements must be calibrated so as to yield a realistic distribution of quantities produced and used. We rely on the 2002 U.S. Agricultural Census (USDA-NASS), the USDA Agricultural Resource Management Survey databases (USDA-ARMS), and weather data from PRISM at Oregon State University to accomplish this objective (see Appendix for further details).

We calibrate the skewness of the firm-specific deviations from the "regional" $\boldsymbol{\alpha_r}$ by fitting a standard Beta distribution to the county-level data of the Census variable "Total sales, Value of sales, number of farms" which serves as a proxy for firm size.[9] The shape parameters are estimated by maximum likelihood, yielding a positive skewed distribution. This is consistent with the higher proportion of small firms observed in each region.

The size of the elements in $\boldsymbol{a_f}$ is tackled by inducing positive rank correlation among the Beta random shocks, such that a firm producing high levels of output is more likely to use greater amounts of inputs.

Finally, to calibrate the unobserved dispersion of $\boldsymbol{\alpha_f}$ from $\boldsymbol{\alpha_r}$, we assume that observed yield dispersion in a region is a function of unobserved technology heterogeneity and observed random weather shocks. If all firms used the same technology, the observed yield variability would come only from weather shocks. At the other extreme, where all firms differ but no weather shocks occur, all yield dispersion comes from heterogeneity across firms. Since it is most likely that reality

[9] The Beta distribution is chosen because it can mimic the different levels of skewness observed in the distribution of these variables at the firm level. Skewness can be manipulated by appropriately choosing its two shape parameters. The support of the standard Beta distribution, the interval [0, 1], only covers positive values.

is somewhere between the two extremes, we estimate the portion of yield variation attributable to heterogeneity across firms. To this end, we use a panel of firm-specific crop yields from USDA-ARMS database and county-specific weather data (growing season precipitation and temperature) from PRISM over five years, and estimate a fixed-effects model to infer the variability across firms that is not due to weather. Details of the estimation are presented in Appendix B.

B. Generation of quasi-fixed netput quantities: $\boldsymbol{K}_{ft}^{*}$

We obtain the vector $\boldsymbol{K}_{f}^{*}$ of quasi-fixed netputs by drawing $R \times F$ Beta distributed random deviates. The Beta distribution is chosen because it can mimic the different levels of skewness observed in the distribution of these variables at the firm level. Because we choose to represent farm size as the quasi-fixed netput, we use the 2002 U.S. Agricultural Census variable "Farms & land in farms, approximate land area" to calibrate the parameters of the Beta distribution for each region.[10] This shows a relative abundance of small-sized farms, implying a positively skewed standard Beta distribution. Region-specific distributions include: $\boldsymbol{K}_{f,r=1}^{*}$ ~ Beta(0.5679, 6.9707); $\boldsymbol{K}_{f,r=2}^{*}$ ~ Beta(0.6026,9.0446); and $\boldsymbol{K}_{f,r=3}^{*}$ ~ Beta(0.4929, 2.9624).

Because both $\boldsymbol{K}_{f}^{*}$ and A_f determine the netput quantities, we generate the vector of quasi-fixed netputs by imposing positive correlation with the production function parameters. We use the method in Iman and Conover (1982) to impose rank correlation.

Next, we generate time variation in each firm's quasi-fixed netput quantity by means of a multiplicative and independent shock centered at one and uniformly distributed. That is, $\boldsymbol{K}_{ft}^{*} =$

[10] It is common practice to include land as a quasi-fixed output.

$\boldsymbol{K}_f^* \epsilon_{ft}$, where ϵ_{ft} ~ Uniform[0.90, 1.10]. The narrow interval implies low variation in firm size over time, which is meant to represent the observed low dispersion over time of aggregate agricultural area in a region.[11]

C. Generation of normalized variable netput prices: $\boldsymbol{p}_{ft}^{**}$ [12]

We generate a set of firm-specific normalized exogenous prices for each region (i.e., prices normalized by the price of the numeraire good). Exogeneity is with respect to the aggregated netput quantity produced, but not with respect to quasi-fixed netput quantities $\boldsymbol{K}_{ft}^*$ and starting technology parameters $\boldsymbol{a}_f^*$. While we acknowledge the existence of price endogeneity, we generate them exogenously in order to have a dataset with minimal sources of noise.

We begin by simulating "national" netput prices to match the properties (mean, standard deviation, and serial autocorrelation) of those found in a time series of normalized futures output prices (soybeans, corn and livestock) from the CME and of normalized input prices (hired labor, energy, chemicals, materials, and capital) from Eldon Ball's (USDA-ERS) dataset. We assume firms base their production decisions on expected output prices and current input prices. The numeraire good is wheat.[13] The choice of the numeraire good, the outputs and the inputs is based on the importance of these goods in the region, and follows previous work with the same datasets by Schuring, Huffman, and Fan (2011).

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[11] This creates, for each time period, a distribution of quasi-fixed netput quantities for each firm that is not necessarily the regional Beta (it is Beta with other parameters), but still maintains the required skewed shape due to the lower dispersion of firm size over time.
[12] Although the actual prices in Eldon Ball's dataset could have been used to analyze the performance of the dual approach in the present study, we use simulated prices because our goal is to provide a general procedure that would allow researchers to generate as many observations (time-periods) as desired
[13] CME future prices are used as proxies of expected prices.

We model normalized netput prices as lognormally distributed and behaving according to AR(1) processes:

$$log(p_{nt}) = \theta_{n0} + \theta_{n1}\, log(p_{n,t-1}) + \zeta_{nt} \quad (7)$$

where "n" indexes netputs and ζ_{nt} is an error term distributed $\mathrm{N}\left(0, \sigma^2_{\zeta_n}\right)$. Parameters θ_n are estimated by OLS regressions using Eldon Ball's dataset. Table 1 shows results for each of the n regressions.

Dropping the "n" subscript to ease notation, taking unconditional expectations in (10) yields $E[log(p_t)] = \theta_0 /(1-\theta_1)$. The variance of the error term in (7) can be calibrated from the observed price variation in Eldon Ball's datasets: $\sigma^2_{log(p)} = \theta_1^2\sigma^2_{log(p)} + \sigma^2_\zeta$ which implies that $\sigma^2_\zeta = (1-\theta_1^2)\sigma^2_{log(p)}$. In this case, we calibrate price variation from a combination of data observed variance and regression results, and not exclusively from the latter.

Table 1. Estimation results of the OLS regression model used to generate normalized random exogenous "national" prices from equation (7).

	$\boldsymbol{n=1}$	$\boldsymbol{n=2}$	$\boldsymbol{n=3}$	$\boldsymbol{n=4}$	$\boldsymbol{n=5}$	$\boldsymbol{n=6}$	$\boldsymbol{n=7}$	$\boldsymbol{n=8}$
	-0.031	-0.065	-0.012	-0.031	-0.001	-0.057	0.041	0.001
	(0.038)	(0.032)	(0.030)	(0.064)	(0.035)	(0.035)	(0.024)	(0.037)
	0.680	0.34	0.67	0.902	0.861	0.60	0.843	0.923
	(0.094)	(0.14)	(0.11)	(0.079)	(0.080)	(0.12)	(0.080)	(0.054)
$\boldsymbol{\sigma^2_\zeta}$	0.0680	0.0342	0.0372	0.0340	0.0439	0.0392	0.0207	0.0237

Note: Standard errors in parenthesis. Number of observations in each regression: 44.

To draw exogenous log-normal netput prices, we fit (7) with the estimated parameters, set $log(p_{s=0}) = \theta_0/(1-\theta_1)$, and take a draw from a $N(0,(1-\theta_1^2)\sigma^2_{log(p)})$ random variable, yielding a netput log-price for each n in the first iteration, i.e. $log(p_{s=1})$. We repeat this procedure

S=10,000 times; we keep the last 50 iterations to form the set of exogenous "national" netput prices and burn the remaining iterations.

Finally, we generate firm-specific netput prices $\boldsymbol{p}_{ft}^{**}$ as deviations from the "national" market price, deviations that are small relative to $\boldsymbol{p}_t^{**}$ to acknowledge the small variability of contemporaneous prices across firms. A regional average is first calculated as $\boldsymbol{p}_{rt}^{**} = \boldsymbol{p}_t^{**} d_r \varepsilon_{rt}$,[14] where d_r is a regional indicator with mean one across regions[15] and ε_{rt} is a mean one symmetric shock distributed as $\varepsilon_{rt} \sim [0.95 + 0.1Beta(2,2)]$. Random variables d_r and ε_{rt} are symmetric and independently distributed. The indicator implies prices of region r are on average $(d_r - 1)$% away from the national average, and the ε_{rt} allows for non-constant deviations over time.

From the regional prices, we generate F firm-specific random prices per region as deviations from the regional average: $\boldsymbol{p}_{ft}^{**} = \boldsymbol{p}_r^{**} \varepsilon_{ft}$, where ε_{ft} is a symmetric mean one shock distributed as $\varepsilon_{ft} \sim [0.80 + 0.40Beta(2,2)]$. Shocks ε_{rt}, ε_{ft}, and d_r are independent. Parameters of the ε_{ft} distribution are calibrated using prices from the USDA-ARMS dataset, such that they yield a coefficient of variation of 0.08. This coefficient of variation is twice as large as the one observed in the USDA-ARMS dataset, which should favor the recovery of the production parameters using the dual approach.

The simulated netput prices are correlated with quantities at the aggregate level, but independent at the firm level. While actual prices received and paid may arguably be correlated with firm size, we assume independence so as to favor parameter identification. Also, observed prices in USDA-ARMS show the majority of firm-level prices are concentrated in four or fewer

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[14] The same procedure and shocks are used for $\boldsymbol{p}_{ft}^{**}$.

[15] The values of d_r are 0.90, 1.00, and 1.10 for regions 1 through 3 respectively.

different clusters in each region; however, we generate a "continuum" of firm-specific prices to favor identification.

D. Profit Maximization Problem

The panel dataset is formed by variable netput quantities and prices, and quasi-fixed netputs: $[\boldsymbol{y}_{ft}^{**}, \boldsymbol{p}_{ft}^{**}, \boldsymbol{K}_{ft}^{*}]$. We solve the problem in (3) with exogenous prices received or paid $\boldsymbol{p}_{ft}^{**}$. These results are used in our exercise of applying duality theory to recover production technology using time-series data whose only source of noise is aggregation across heterogeneous firms. This constitutes the minimum possible noise when interested in applying duality theory with time series. The potential of the generated dataset is not restricted to this application; future research may use it to address, for example, the analysis of duality theory with cross-sectional data, or the performance of duality theory when data available (both time-series or cross-sectional) embed sources of noise that depart from duality theory assumptions.

Under the normalized quadratic production function $\boldsymbol{G}(\boldsymbol{y}_{ft}^{**}, \boldsymbol{K}_{ft}^{*}; \boldsymbol{\alpha}_f)$ in (8), the FOCs are:

$$\boldsymbol{p}_{ft}^{**} - A_{1f} - A_{11f}\boldsymbol{y}_{ft}^{**} - A_{12f}\boldsymbol{K}_{ft}^{*} = 0 \tag{8}$$

This system is jointly solved for the vector of optimal variable netput quantities $\boldsymbol{y}_{ft}^{**}$ as a function of the vector of variable netput prices $\boldsymbol{p}_{ft}^{**}$**,** the vector of quasi-fixed netput quantities $\boldsymbol{K}_{ft}^{*}$, and the production parameters $\boldsymbol{a}_f^{*}$. The solution is:

$$\boldsymbol{y}_{ft}^{**}(\boldsymbol{p}_{ft}^{**}, \boldsymbol{K}_{ft}^{*}; \boldsymbol{a}_f^{*}) = A_{11f}{}^{-1}(\boldsymbol{p}_{ft}^{**} - A_{1f}A_{12f}\boldsymbol{K}_{ft}^{*}) \tag{9}$$

This produces a panel dataset of production plans for the $(R \times F)$ firms over T time periods that can be used to recover production parameters using time-series or cross-section. We denote this dataset as follows:

$$[\boldsymbol{y}_{ft}^{**}, \boldsymbol{p}_{ft}^{**}, \boldsymbol{K}_{ft}^{*}] \quad (10)$$

Data for Estimation

In agreement with this study's objective of using the dual approach with time-series data, before estimation we proceed to aggregate the sub-vector $[\boldsymbol{y}_{ft}^{**}, \boldsymbol{p}_{ft}^{**}, \boldsymbol{K}_{ft}^{*}]$ across the F = 10,000 heterogeneous firms for each of the T = 50 periods of time, as if data came from a "single firm." This aggregation is performed on the data described in (10).[16]

For netput quantities, we aggregate by adding across firms since they are homogeneous commodities. The n^{th} netput price at period t (p_{nt}) is a quantity-weighted average of the firm-specific netput prices.

$$\begin{aligned} \boldsymbol{y}_t^{**} &= \textstyle\sum_f \boldsymbol{y}_{ft}^{**} \\ \boldsymbol{K}_t^{*} &= \textstyle\sum_f \boldsymbol{K}_{ft}^{*} \\ p_{nt}^{**} &= (y_{nt}^{**})^{-1} \textstyle\sum_f p_{nft}^{**} y_{nft}^{**} \end{aligned} \quad (11)$$

The literature on properties for consistent aggregation is vast, including Gorman (1968), Richmond (1976), Stoker (1984), and applications in agricultural production economics by Chambers (1988), Chambers and Pope (1991, 1994), Davis (1997), and LaFrance and Pope (2008). However, a linear aggregation is sufficient to achieve the results intended here. [17] The time-series dataset used in estimation is denoted as follows:

$$[\boldsymbol{y}_{t}^{**}, \ \boldsymbol{p}_{t}^{**}, \boldsymbol{K}_{t}^{*}] \quad (12)$$

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[16] If the objective were to study empirical properties of duality under a cross-sectional dataset, we would have taken one year of the panel and conducted the analysis without aggregating across firms. This is left for future research.
[17] In future research, it would be interesting to explore the robustness of the results to alternative aggregation methods for the generated data.

The dataset in (12) includes all $n = 8$ netput quantities and prices, and $m = 1$ quasi-fixed netput. Variable netput prices are exogenous from quantities, but have serial autocorrelation. This aggregation results in a dataset of 50 observations for each variable per region. To avoid the addition of another source of noise coming from heterogeneous technology across regions, we select region 1 to conduct the estimation, and compare results with the initial production parameters of that same region.

Estimation

We approximate the restricted profit function $\pi_R(\boldsymbol{p}, \boldsymbol{K})$, which solves problem (3), by the following normalized quadratic flexible functional form:

$$\pi_R(\boldsymbol{p}, \boldsymbol{K}; \boldsymbol{\beta}) = \boldsymbol{p}'B_1 + \boldsymbol{K}'B_2 + \frac{1}{2}\boldsymbol{p}'B_{11}\boldsymbol{p} + \boldsymbol{p}'B_{12}\boldsymbol{K} + \frac{1}{2}\boldsymbol{K}'B_{22}\boldsymbol{K} + \boldsymbol{p}'\boldsymbol{\kappa} \qquad (13)$$

where B_1 and B_2 are $(n \times 1)$ and $(m \times 1)$ vectors of β_i coefficients, B_{11} is a symmetric $(n \times n)$ matrix, and B_{12} and B_{22} are $n \times m$ and $m \times m$ matrices. Submatrices $B_{11},\ B_{12}$, and B_{22} form a symmetric $\left((n + m) \times (n + m)\right)$ matrix B of β_{ij} coefficients, which in the case of the normalized quadratic profit function is exactly the Hessian matrix with respect to $(\boldsymbol{p}, \boldsymbol{K})$. All β_i and β_{ij} coefficients collectively form the set $\boldsymbol{\beta}$. The error structure $\boldsymbol{p}'\boldsymbol{\kappa}$ is consistent with the McElroy (1987) additive general error model (AGEM) applied to the case of profit functions. The $(n \times 1)$ vector of random variables $\boldsymbol{\kappa}$ is jointly normally distributed with mean equal to an $(n \times 1)$ vector of zeros and an $(n \times n)$ covariance matrix $\boldsymbol{\Sigma}_\kappa$. This covariance matrix induces contemporaneous correlation between the equations. Also, the DGP of netput prices—both exogenous and endogenous—was constructed as an AR(1) process, implying serial autocorrelation in the independent variables that needs to be accounted for in the estimation.

We derive the set of input demands and output supplies by taking first derivatives of (13) with respect to netput prices (Hotelling's lemma), yielding the system to be estimated:

$$\boldsymbol{y}(\boldsymbol{p}, \boldsymbol{K}; \boldsymbol{\beta}) = B_1 + B_{11}\boldsymbol{p} + B_{12}\boldsymbol{K} + \boldsymbol{\kappa}. \tag{14}$$

We conduct estimation by iterated SUR, which converges to maximum likelihood, and is the most common method employed in empirical studies based on duality theory. We impose symmetry cross-equation restrictions ($\beta_{ij} = \beta_{ji}, i \neq j$) in matrix B_{11}. We do not estimate the parameters of the profit function because the parameters needed to evaluate the production parameters of interest are present in the demands and supplies.

It has to be noted that the core of the empirical applications of Duality theory consists of performing an econometric estimation along the lines described in the preceding two paragraphs, plus a thorough interpretation of the results. In contrast, what is unique about this study is that we not only employ these widely accepted estimation procedures, but also generate the pseudo-data and compare the estimated parameters with their "initial" counterparts.

We treat mean-independence violations in estimation by noting that an inspection of the autocorrelation and partial autocorrelation functions of the time series suggests first differentiation of the data for estimation.[18] This is a consequence of the DGP of price data as AR(1) processes.

The estimated values of matrix B_{11} and vector B_{12} are the focus of our attention; they are, respectively, the first derivatives (marginal effects) of netput quantities with respect to prices and quasi-fixed netputs, and therefore they are the base to construct the estimated profit function

[18] This implies that the elements of the vector B_1 are not identified. However, knowledge of their values is not required because they do not enter the formulae of output supply and input demand elasticities with respect to prices, only elements of B_{11} do (e.g., the elasticity of the i^{th} netput with respect to the j^{th} price is computed as $[B_{11}]_{ij}(\bar{p}_j/\bar{y}_i)$, where $\bar{p}_j$ and $\bar{y}_i$ are the means of the observed sample values of the j^{th} price and the i^{th} netput quantity).

Hessian matrix $[\breve{B}]$ and the elasticities matrix of netput quantities with respect to own price, cross prices, and quasi-fixed netputs $[\breve{E}]$. As described in Figure 2, we obtain matrix $[\breve{B}]$ from estimation using the described data transformed to time series. This matrix is then transformed into an elasticity matrix in a straightforward way (see footnote 18).

In order to compare estimated elasticities with starting values, we proceed as follows. We begin from the starting and known firm-specific production function Hessian matrix $[A]_f$ and convert it into the corresponding profit function Hessian $[B]_f$ using Lau's Hessian identities. We further transform the starting profit function Hessian into a matrix of own- and cross-price elasticities and quasi-fixed elasticities of netput quantities $[E]_f$. Finally, as indicated in Figure 2, we compare the distribution of the initial $[E]_f$ versus the estimated values ($[\breve{E}]$) to evaluate how precisely we recover the starting price and quasi-fixed netput elasticities under duality theory. Note that this comparison implies that the initial values are represented by a distribution of each firm's initial parameters, whereas the estimated values consist of a point estimate and its confidence interval.

Results

Figure 3, Figure 4, and Table 2 summarize the results from estimating output supplies and input demands parameters in (14). In Figure 3 we show how the estimated own- and cross-price elasticities of netput quantities ($\breve{\boldsymbol{E}}_{\boldsymbol{ij}}$)[19] compare with the distribution of initial firm-specific elasticities, the mean of the distribution ($\overline{\boldsymbol{E}}_{\boldsymbol{ij}}$) and its median ($\overline{\overline{\boldsymbol{E}}}_{\boldsymbol{ij}}$), for the 64 entries of the 8×8 elasticity matrix. The vertical axis represents the mean of the distribution of initial elasticities, and

[19] The estimated parameters are not reported for reasons of space, but they pass tests for global curvature (as the estimated matrix B_{11} is positive semi-definite), and monotonicity (because the estimated output supply and input demand quantities are positive when evaluated at data means). We also tested for monotonicity at every data point and found that it is violated in only three out of the 50 years (and when it does, it is violated for only one netput).

the horizontal axis shows descriptive statistics of the distribution of initial elasticities: the 90% highest probability density interval of the initial distribution (the horizontal line), the median (filled square) of the distribution, the SUR estimated elasticity (circle), and the corresponding 95% confidence interval (vertical lines). We set the 45º line to intersect each distribution (horizontal line) at its mean. The median (filled square) is to the left or to the right of the mean depending on the skewness of the distribution. The elasticity point estimates (circle) and their 95% confidence intervals (vertical lines) are in all cases within the support of the initial distribution.

This implies that estimation with a dataset constructed as the aggregation across heterogeneous firms (as if it belonged to a representative firm) is able to recover elasticities that are not only within the relevant range of the distribution but also fairly close to the median and the mean.[20]

A second conclusion arises by noting that the point estimates are closer to the median of the distribution than to the mean. In other words, the representative firm is better described by the median of the distribution than the mean. The root mean squared error (RMSE) helps illustrate this conclusion. The RMSE is the average difference between each entry of the estimated elasticity matrix versus its corresponding simulated elasticity, expressed in elasticity units. We show two alternative values to describe the simulated elasticity: the median of the firm-specific elasticity distribution and its mean. When compared to the median of the distribution, the RMSE is:

$$RMSE = \left[\frac{1}{64 \times S}\sum_{i}\sum_{j}\sum_{s}\left(\bar{\bar{E}}_{ij,s} - \check{E}_{ij,s}\right)^{2}\right]^{1/2} \tag{15}$$

[20] A nonlinear aggregation method of heterogeneous firms, as those discussed by Chambers (1988), is expected to achieve more precise results, but the linear one performed here is sufficient for the objectives of this paper because it recovers production parameters with enough precision.

where S = 10,000 is the number of draws from the limiting distribution of the SUR parameter estimates and the subscript s indicates the s^{th} draw of the ij^{th} parameter, with $i, j = 1,2, \dots ,8$. For comparison with the mean we substitute $\bar{\bar{E}}_{ij}$ by $\bar{E}_{ij}$. The RMSE averages over all of the $64 \times S$ squared differences. We also provide a measure of its dispersion by calculating the standard deviation of these $64 \times S$ values before averaging over them. The RMSE standard deviation contains two sources of variation or error. One is due to the SUR estimation error within each of the 64 parameters, and the other is associated with the variation of the difference between the estimated and the starting value of the elasticity across the 64 parameters.

As shown in Table 2, RMSE is 0.029 in the case of the median and almost the double (0.057) for the mean. To put these values into perspective, we calculate the percentage deviation of the RMSE with respect to the descriptive statistics of the starting distribution of elasticities. Relative to the median it yields a difference of 9.2% and, as expected, it is higher relative to the mean, 17.3%.

Table 2. Comparison of estimated elasticities versus moments of the distribution of initial elasticities.

Elasticities with respect to		**Moment of Initial Distribution**	
		Median	Mean
	RMSE	0.029	0.057
	Std. error	0.051	0.127
	% deviation	9.2	17.3
	RMSE	0.036	0.040
	Std. error	0.054	0.050
	% deviation	9.0	9.6

The RMSE standard deviation is 0.051 for the median and 0.127 for the mean. Given that the SUR estimation provides only a minor source of error (because the point estimates are all highly significant due to the use of a data with only minor sources of noise),[21] the majority of the RMSE standard deviation is attributed to the deviations between the estimated and the initial value across elasticities. Since the skewness of initial production parameters drives the skewness of netputs quantities, and here firms are linearly aggregated, these deviations would most likely decrease under nonlinear aggregation.

A third conclusion from Figure 3 is that the estimated standard errors of the SUR estimators exhibit a substantial downward bias, i.e., the SUR elasticity estimates are far less precise than indicated by their SUR estimated standard errors. The evidence for this result is that for the majority of the elasticities shown in Figure 3, the population mean is outside of the respective 95% confidence interval. In fact, only 10.9% (= 7/64) of the 95% confidence intervals include the corresponding means, whereas by construction one would expect approximately 95% of such intervals to do so.[22] If each confidence interval comprised the mean with 95% probability, and intervals were independent, the probability of having at most 7 out of 64 means lie inside their respective confidence intervals would be essentially 0.

Figure 4 illustrates the estimated results of the eight netput quantity elasticities with respect to the quasi-fixed input. The SUR estimated elasticities (circles) are within the interval of simulated elasticity distribution for all cases and, similar to the variable netputs case, closer to the median of the distribution than to its mean. As Table 2 indicates, the RMSE is 0.036 in the case of the median, and 0.040 for the mean. The size of the RMSE standard deviation also suggests high variation (of their dispersion relative to the initial value) across the 8 elasticities. Our estimated elasticities are

[21] Results are available from the authors.
[22] Even if one were to argue that the SUR elasticity estimate represents the underlying population median rather than the mean, the conclusion would apply because only 23.4% (= 15/64) of the 95% confidence intervals include the corresponding medians.

9.0% apart from the median absolute value of the initial elasticity and 9.6% from the mean absolute value. Figure 4 also shows that the downward bias found in the estimated standard errors of the SUR estimators of the price elasticities also applies to the quasi-fixed input elasticities, as just 25% (= 6/8) of the 95% confidence intervals include the corresponding population means.

Conclusions

The dual relationship between the production function and the profit or cost function established by the Neoclassical theory of the firm has been widely applied in empirical work with the objective of obtaining, among others, price elasticities, substitution elasticities, and return to scale estimates. This empirical method, usually referred to as "the duality theory approach," has the advantage of providing the mentioned features of the production function using market data on input and output prices and quantities, without the requirement of explicitly specifying the parametric form of the production function. However, the duality theorem requires a set of assumptions about the DGP which are unlikely to hold in practice; or in other words, the market data typically employed in this type of studies bear levels of noise that prevent the theorem from holding exactly. If this is the case, the estimated technology need not provide a good estimate of the starting underlying technology.

The impact of random noise on the empirical performance of the dual approach is an area that has received scant attention in the literature. The present paper aims at contributing to this area of inquiry by showing how to generate a pseudo-dataset that replicates key aspects of U.S. agriculture. Model parameters are calibrated using datasets (both time-series and cross-sectional) widely employed in empirical applications. We start by selecting a parametric form of the production technology and choosing its set of parameter values. By means of Monte Carlo simulations, we generate observations of quantities and normalized netput prices such that they are consistent with those found in data on U.S. agriculture. More precisely, we compute a panel of production and price data for successive periods of time, originated from a population of technologically heterogeneous firms that belong to different regions.

We illustrate the usefulness of the simulated dataset by applying it to analyze the ability of the duality approach to recover the underlying production technology when the only source of deviation from the basic tenets of the theory is the aggregation across heterogeneous firms. Estimated parameters (and resulting elasticities) come from applying standard econometric methods to a system of input demands and output supplies with the simulated data. Because the initial parameters are known from the outset, we can judge the degree to which the econometric approach applied on the dual problem recovers these parameters. Comparison between starting and recovered parameters relies on the use of Hessian identities. Our exercise, restricted to the dual-primal direction, can be complemented in future research with another carried out on the primal-dual direction.

While the main results of this applications are as expected, that is, the dual approach applied to a time-series coming from aggregation across technologically heterogeneous firms is able to recover elasticities that are not only within the support of the distribution of initial elasticities, but also considerably close to the mean and median of such distribution, it is also found that the estimated standard errors of the SUR elasticity estimators exhibit a substantial downward bias (i.e., the SUR elasticity estimates are far less precise than implied by their SUR estimated standard errors). More importantly, this simulated dataset allows us to set a solid base to study any agricultural problem that requires knowing the underlying production parameters that generated the data. For example, a more in-depth understanding of the empirical properties of duality theory can be achieved with this pseudo-dataset, because it is straightforward to calibrate and incorporate other sources of observable and unobservable noise commonly found in datasets used by practitioners, such as uncertainty, prediction errors, omitted variables, netput aggregation, and endogeneity. This pseudo-dataset may also be used to investigate whether increasing the level of noise impacts linearly or nonlinearly the ability to recover the underlying technology parameters, or whether there is a noise threshold above which it is not worth applying the dual approach. Another examples are the evaluation of alternative functional forms, or the various methods used to estimate technical change (e.g., the pseudo-dataset can be generated such that firm parameters change over time due to the evolution of technology, and study how precise are these methods in identifying and recovering the evolution of technology when it is explicitly accounted for in the estimation). Such analyses are complex and beyond the scope of the present study, which is meant to provide a building block for them. However, future research should address it to yield a better understanding of the empirical properties of duality theory.

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Appendix A. Generation of the firm-specific set of initial production parameters $\boldsymbol{a}_{\boldsymbol{f}}^{*}$

We start from the "generic" set $\boldsymbol{a}$ which is composed by the vectors A_1, A_2, A_{11}, A_{12}, A_{21}, A_{22}, and matrix A_{11}, where

$$A_1 = [0.007 \quad 0.007 \quad 0.004 \quad -0.059 \quad -0.314 \quad -0.028 \quad -0.017 \quad -0.081]'$$

$$A_2 = -70.620$$

$$A_{11} = \begin{bmatrix} 1.569 & 1.062 & 0.027 & 2.245 & 1.986 & 1.378 & 0.742 & 3.120 \\ 1.062 & 1.899 & 0.051 & 3.025 & 2.840 & 1.985 & 1.143 & 4.435 \\ 0.027 & 0.051 & 3.154 & 4.059 & 1.030 & 1.530 & 3.356 & 1.646 \\ 2.245 & 3.025 & 4.059 & 16.789 & 7.045 & 5.359 & 7.293 & 10.778 \\ 1.986 & 2.840 & 1.030 & 7.045 & 17.592 & 4.210 & 2.169 & 9.662 \\ 1.378 & 1.985 & 1.530 & 5.359 & 4.210 & 5.624 & 3.481 & 6.311 \\ 0.742 & 1.143 & 3.356 & 7.293 & 2.169 & 3.481 & 7.016 & 3.539 \\ 3.120 & 4.435 & 1.646 & 10.778 & 9.662 & 6.311 & 3.539 & 19.629 \end{bmatrix}$$

$$A_{12} = (A_{21})' = [5.493 \quad 8.967 \quad 8.221 \quad 35.195 \quad 36.758 \quad 19.172 \quad 20.705 \quad 38.203]$$

$$A_{22} = 150.640$$

These values are based on profit function estimated parameters $\boldsymbol{B}_{\boldsymbol{ij}}$ found in literature using Eldon Ball's dataset (Schuring, Huffman, and Fan 2011). We transform the original estimates to meet the desired convexity properties and convert them to production function parameters using Hessian identities. This provides us with a first approximation of the parameters' values.

Vectors $A_{1,r}$, $A_{2,r}$, $A_{1,f}$ and $A_{2,f}$: For r = {1,2,3}, we obtain the regional vectors $A_{1,r}$ and $A_{2,r}$ by respectively affecting each entry of A_1 and A_2 by independent multiplicative shocks $\nu_a \sim$ Uniform[0.60, 1.40]. Then, the farm-specific vectors within each region $A_{1,f}$ and $A_{2,f}$ are obtained from each regional value also by inducing variation with correlated and multiplicative shocks μ_a distributed as Beta. These shocks determine the size, dispersion and skewness of the netput quantities produced, so they need to be calibrated accordingly. To control for the skewness, we use the county-level variable "Total sales, Value of sales, number of farms" of the 2002 U.S.

Agricultural Census as a proxy of firm size, to fit a standard Beta distribution for each region; the results are Beta(0.3062, 2.5654) for region 1, Beta(0.2810, 2.4012) for region 2, and Beta(0.3315, 2.1364) for region 3. To calibrate the desired variability of the firm-specific parameters, we modify the interval widths of the Beta distributions to [0.90, 1.40], [0.90, 2.00], and [0.90, 4.20] for regions 1 through 3 respectively, so the Beta distributions replicate the coefficient of variation of the technology parameters estimated by the fixed-effects regression using USDA-ARMS and PRISM datasets described in the text. Because parameters determine firm size, we impose a positive correlation of 0.9 between the shocks, so that firms producing high output quantities also use more inputs. In all cases, correlation is imposed by the method in Iman and Conover (1982).

Matrices $A_{11,r}$ *and* $A_{11,f}$: We generate the inverse of the regional and firm-specific matrices $A_{11,r}$ and $A_{11,f}$, because the latter is the one entering the FOCs of the firm's optimization problem. First, we perturb each entry of an upper triangular matrix C_r representing the Cholesky factorization of the "generic" positive-semidefinite matrix $(A_{11})^{-1}$, such that $\left(A_{11,r}\right)^{-1} = C_r' C_r$. This guarantees the matrices of interest are positive-semidefinite in each iteration. The regional deviations come from using an independent and multiplicative shock $\nu_b \sim$ Uniform[0.70, 1.30]. Then, to obtain the firm-specific submatrices $\left(A_{11,f}\right)^{-1}$ in each region, we induce variation on the Cholesky factors of $\left(A_{11,r}\right)^{-1}$ with correlated and multiplicative Beta shocks μ_b with shape parameters mentioned in the previous paragraph, but over the intervals [0.90, 1.20], [0.90, 1.60] and [0.80, 2.60] for regions 1, 2, and 3 respectively. Again, we set the interval width so that the coefficient of variation of the parameters matches that from the fixed effects regression for each region, as explained in Appendix B. Also, we impose positive correlation among the parameters of the matrix to control for firm size.

Vectors $A_{12,f}$ and $A_{22,f}$: Similar to the case of matrices $A_{11,r}$ and $A_{11,f}$, we construct these vectors, as well as the "generic" vectors A_{12} and A_{22}, starting from the "generic" profit function parameters B_{12} and B_{22}, and using Hessian identities in (4). This is done not only to guarantee theoretically consistent values of the vectors of interest, but also because profit function parameters are readily available in the literature. We respectively shock each entry of B_{ij} by independent multiplicative deviations $\nu_c \sim$ Uniform[0.95, 1.05], obtaining regional $B_{ij,r}$. The corresponding firm-specific values ($B_{ij,f}$) within a region come from deviations of the regional $B_{11,r}$, $B_{12,r}$ and $B_{22,r}$ by means of multiplicative and correlated shocks Beta μ_c, and then transformed into $A_{12,f}$ and $A_{22,f}$ using the Hessian identities in (4). Note that in this process we do not directly generate regional vectors $A_{12,r}$ and $A_{22,r}$. The shape parameters of the Beta distribution are the ones stated above, and the intervals for $B_{12,f}$ are set at [0.90, 2.00], [0.90, 2.20], and [0.80, 3.60] for each region, and at [0.90, 1.10] for all regions in the case of $B_{22,f}$. The narrow interval in the latter case is due to the fact that enough variation is already induced on $A_{22,f}$ by $B_{11,f}$, $B_{12,f}$ and $B_{22,f}$ through the Hessian relationship. Finally, we impose positive correlation between the entries of $A_{12,f}$ and $A_{22,f}$ to take care of firm size.

We calibrate the width of the Beta intervals enumerated above by trial and error such that they yield a set of firm-specific production parameters $\boldsymbol{a}_f^*$ in each region whose coefficient of variation is consistent with $\breve{b}_{0c}$ estimated with the fixed-effects model. These are 0.06, 0.17, and 0.43 for regions 1, 2, 3, respectively, as shown in table A1.

Appendix B. Estimation of firm's unobserved heterogeneity

Yields are specified as a function of a county-specific constant (the fixed effect) representing the average county's technology, and cumulative precipitation and average temperature over the growing season, and we assume the constant is correlated with the weather variables. This allows us to isolate the "between" effects (i.e., the variation in yields across counties not attributable to weather) from the "within" effects (i.e., the variation in yields within a county over time).

Firm-level yields are specified as follows:

$$y_{ft} = b_{0c} + b_1 W_{1ct} + b_2 W_{2ct} + b_3 D_{1t} + \cdots + b_6 D_{4t} + \epsilon_{1ft} \tag{16}$$

where c, f and t index counties, firms, and time respectively. Variables W_1 and W_2 are precipitation and temperature, respectively, for the county, and D_1 through D_4 are year dummy variables (2001 through 2004 respectively, with year 2000 as the base). The parameter b_{0c} represents county-level technology and is the focus of our interest. Because we presume it to be correlated with weather variables, we estimate a fixed-effects model where parameters b_1 through b_6 are estimated by demeaning the data (means taken for each county and over time), resulting in the following model (Greene 2003):

$$\ddot{y}_f = b_1 \ddot{W}_{1c} + b_2 \ddot{W}_{2c} + b_3 \ddot{D}_1 + \cdots + b_6 \ddot{D}_4 + \epsilon_{2f} \tag{17}$$

with " ¨ " indicating demeaned variables, estimated by OLS. The county-specific parameter b_{0c} is then recovered by calculating the following equation:

$$\check{b}_{0c} = \bar{y}_c - \hat{b}_1 \overline{W}_{1c} - \hat{b}_2 \overline{W}_{2c} - \hat{b}_3 \overline{D}_1 - \cdots - \hat{b}_6 \overline{D}_4 \tag{18}$$

where the " ¯ " indicates means over time (used in demeaning the model) and the " ^ " indicates the point estimate of the parameters. Table A1 provides estimation results.

Finally, the coefficient of variation of $\check{b}_{0c}$, representing variation across counties, serves to calibrate the unobserved dispersion of the production parameters $\boldsymbol{a_f}$ around the regional mean $\boldsymbol{a_r}$

that are not attributable to weather changes.[23] Note that this coefficient of variation represents the variation across counties of the fitted production coefficients, rather than the estimation standard error of the parameter.

Table A1. Parameter estimates of fixed effects model, equation (17), to calibrate production function parameter variation, and realized weather shocks on netput quantities.

Dependent variable: $\ddot{y}_f$	**Region 1**	**Region 2**	**Region 3**
Explanatory variables	**Parameter estimates: $\boldsymbol{b_i, i = 1, \ldots, 6}$**		
	-0.0002	0.0040	0.0019
	(0.0008)	(0.0007)	(0.0014)
	-0.320	-0.041	0.003
	(0.032)	(0.023)	(0.036)
	5.72	-12.52	4.760
	(7.24)	(2.05)	(3.074)
	-17.57	0.23	-7.90
	(5.16)	(1.65)	(3.21)
	-17.58	2.73	-2.55
	(4.16)	(1.88)	(2.79)
	8.74	1.63	27.99
	(4.13)	(1.81)	(4.11)
Firm heterogeneity contribution to yield variation: CV($\breve{b}_{0c}$)	0.0578	0.1702	0.4276
Weather variables contribution to yield variation (CV)	0.0726	0.1263	0.4040

Variable $\ddot{y}_f$ denotes demeaned farm-specific crop yields. Accent character " ¨ " represent a demeaned variable. Standard errors in parenthesis.

[23] We calibrate the production parameter variation equal to variation between counties, as opposed to between firms. Firstly, we do not have firm-specific weather data to calculate the between firms effects. Secondly, in a given region the data are likely to have smaller variation at the county (and more aggregated) level than at the firm level, favoring parameter recovery.